Symmetric Positive Equilibrium Problem: A Framework for Rationalizing Economic Behavior with Limited Information: Comment

Wolfgang Britz, Thomas Heckelei, and Hendrik Wolff

In a recent contribution to this journal, Paris suggests a framework which extends positive mathematical programming (PMP)—a widely used calibration methodology for agricultural supply models—to a symmetric positive equilibrium problem (SPEP). He stresses three main contributions: (1) The PMP methodology is modified to incorporate more than one observation on production programs; (2) A solution to the "self-selection problem" with respect to the choice of crops produced by each farm is provided; (3) "Limiting inputs" are no longer considered fixed quantities as in PMP.

We address several conceptual concerns with respect to the SPEP methodology and the presented application. We consider these to be substantial enough to question Paris' claim to present "... a general framework of analysis that is capable of reproducing economic behavior in a consistent way ..." (p. 1049). Our discussion is structured along Paris' presentation: The next three sections represent the core of the comment and deal with the methodology itself. They refer to the three phases of SPEP: (i) recovery of unknown variable marginal costs and shadow prices of limited resources, (ii) use of these results to specify data constraints and parameter supports for generalized maximum entropy (GME) estimation of a cost function, and (iii) definition of a simulation model. Finally, concluding remarks are made regarding the application of SPEP to an analysis of the EU Common Agricultural Policy (CAP) based on Italian farm data. Throughout the comment we use the same mathematical notation as Paris and refer to his equation numbers to facilitate comparison.

Phase 1: Estimation of Marginal Costs

Two alternative ways to recover marginal cost and shadow prices of limiting inputs are suggested for Phase 1 depending on the number of limiting inputs:[1]

The first alternative is equivalent to the typical PMP procedure. Equations (1)–(4) (or equations (8)–(10) for the sample LP problem) maximize overall gross margin, $(\mathbf{p}_n - \mathbf{c}_n)'\mathbf{x}_n$, subject to land availability and calibration constraints restricting the optimal production quantities to be less than or equal to observed quantities. The model's solution implies a shadow value of land, y_n, that is equal to the gross margin of the least profitable of the produced crops per unit of land, i.e., equal to $\min_j[(p_{jRn} - c_{jRn})/a_{jRn}]$. For all observed cropping activities, the shadow values of the calibration constraints, $\boldsymbol{\lambda}_{jn}$, are equal to $p_{jn} - c_{jn} - y_n \cdot a_{jn}$.[2] Consequently, the values required in Phase 2 can be calculated analytically as long as only one limiting input exists. More importantly, "the marginal cost of limiting inputs, $(\mathbf{A}'_{Rn}y_n)$ and $(\mathbf{A}'_{NRn}y_n)$, and the variable marginal cost associated with output levels, $(\boldsymbol{\lambda}_{Rn} + \mathbf{c}_{Rn})$ and $(\boldsymbol{\lambda}_{NRn} + \mathbf{c}_{NRn})$," (p. 1051) are arbitrary outcomes with potentially significant influence on parameter estimates in Phase 2 through the data constraints (25)–(28). We consider them "arbitrary" in the sense that the combination of variable marginal cost and shadow prices is implied by this form of linear programming model, which is different from the ultimate simulation model used in Phase 3. Thus, other optimization

Wolfgang Britz is a researcher at the Institute for Agricultural Policy, Market Research and Economic Sociology of Bonn University; Thomas Heckelei is an assistant professor, IMPACT Center/Department of Agricultural and Resource Economics; and Hendrik Wolff is a graduate student, Department of Agricultural and Resource Economics, Washington State University.

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[1] In the article, the "extension of SPEP to several limiting inputs" is actually given under Phase 2, but since the presented equilibrium problem is supposed to replace models (1)–(4) of Phase 1 we include it in this section.

[2] This implies a zero value of λ_{jRn} for the least profitable crop.

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models exist that imply different shadow prices and variable marginal cost, for example, Paris' own alternative suggestion for Phase 1.[3]

The alternative for Phase 1 suggested by Paris is described by equations (36)–(45) and motivated as an "extension of SPEP to several limiting inputs" (p. 1054). It can be easily verified that this "equilibrium problem" is nothing but the Karush-Kuhn-Tucker conditions of the following maximization problem:

$$(1) \qquad \max_{\mathbf{x}_n \geq \mathbf{0}, \boldsymbol{\beta}_n \geq \mathbf{0}} \{\mathbf{p}_n'\mathbf{x}_n - \mathbf{c}_n'\mathbf{x}_n + \mathbf{r}'\boldsymbol{\beta}_n\}$$

$$\text{subject to}$$
$$\mathbf{A}_n\mathbf{x}_n + \boldsymbol{\beta}_n = \mathbf{b}_n[\mathbf{y}_n]$$
$$x_{jn} \leq x_{jRn}(1+\varepsilon)[\lambda_{jRn}] \, \forall \, x_{jRn} > 0$$
$$x_{jn} \leq 0[\lambda_{jNRn}] \, \forall \, x_{jRn} = 0.$$

Apart from being defined for several limiting inputs, this formulation reveals another difference in assumption between Paris' equilibrium problem (36)–(45) and the first model described by (1)–(4). The shadow values of resources $\mathbf{y}_n$ are now lower bounded by the exogenous leasing rates $\mathbf{r}$. Keeping $\boldsymbol{\beta}_n$ nonnegative implies that farmers have the opportunity to rent out resources if their marginal profit lies below $\mathbf{r}$, but cannot lease resources for own use if their marginal profit is above $\mathbf{r}$.[4]

Paris does not give an economic or empirical motivation for this rather unusual specification but instead argues for a modification of the standard PMP procedure, because "the specification given in (1)–(4) with multiple constraints on resources produces a vector of dual variables $\mathbf{y}_n$ that contains some zero elements" (p. 1054). That is not true in general since all elements of $\mathbf{y}_n$ can still be positive. In fact, they will be positive if the data imply binding resource quantities, i.e., if the sum of observed resource allocations over all crops is equal to $\mathbf{b}_n$, *and* if all observed gross margins are positive. Failure of either one of these two conditions, however, would also yield a zero dual value in the single resource case.[5] We do not see any principle difference with respect to the two specifications that is related to the number of limiting resources. Note that both suggested specifications for the first phase of SPEP imply profit-maximizing behavior—in contrast to the simulation model in Phase 3.

The last issue regarding Phase 1 we would like to address concerns the treatment of missing data on prices and costs for crops not produced by a farm. The author claims that using average sample information from farms producing these crops "relaxes the distributional assumptions traditionally required for implementing a self-selection framework" (p. 1060). Paris' approach implies a distribution of costs and prices across the sample with a spike at the average value. We consider this a much stronger assumption than the normality assumption implied by Heckman's procedure. A consequence for the later application to Italian farm data is that about 50% additional "observations" are introduced into the sample, including some extreme cases such as potatoes where only three out of thirty-seven farms actually produce the crop.

Phase 2: Parameterization and Estimation

For the case of the aggregate "sample" model, Paris parameterizes marginal cost equations (13) and the "implicit supply of the single limiting input" (p.1052), (15), in such a way that the integration to a cost function results in (17), $C(\mathbf{x}, y) = \mathbf{x}'\mathbf{Q}\mathbf{x}/2 + ysy/2 + \mathbf{x}'\mathbf{H}y$.[6] The nature of this "total variable cost function" (p. 1050) is not clear to us. Apparently, its derivative with respect to the input price y will result in a "derived demand of limiting inputs" (p. 1052). This seems analogous to a dual cost function derived from a cost minimization problem with given output quantities and input prices. However, $C(\mathbf{x}, y)$ is not homogeneous of degree one in input prices, which contradicts this interpretation. Furthermore, the Cholesky factorization in (18) forces $C(\mathbf{x}, y)$ to become *convex* in y[7] instead of concave as required for a regular cost function. Since $C(\mathbf{x}, y)$ violates regularity conditions for a standard dual cost function (see Chambers: 52ff) it must have a different underlying rationale, which we cannot infer from the text.

Our confusion is furthered by the fact that the left-hand sides of equations (13) and (14) do not include the opportunity cost of limiting

[3] For a comparison between the shadow values generated by linear and quadratic programming models, see Heckelei and Wolff.

[4] This behavioral restriction of Phase 1 is inconsistent with the simulation model in Phase 3 where farmers can augment their resources $\mathbf{b}_n$.

[5] Using yearly observations on yields, inputs, and prices implies that observed gross margins might significantly deviate from expected values. If the gross margins of activities with nonzero levels are negative in case of model (1)–(4) or imply marginal resource profitability below $\mathbf{r}$ in case of models (36)–(45), the calibration constraints (3) and (37) must also enter as "greater or equal" constraints with negative perturbations. Otherwise the optimal solutions of these activities are zero and violate the calibration property.

[6] The specification for the single farm is equivalent.

[7] Since s lies on the diagonal of a positive semi-definite matrix.

resources from Phase 1, $\mathbf{A}'y$ or $\mathbf{A}'_n y_n$. Since the derivative of $C(\mathbf{x}_n, y_n)$ with respect to y results in "derived demand of limiting inputs," we assumed that those resources are to be considered variable inputs and consequently contribute to marginal costs. In fact, the appearance of the input price y in the functional representation of marginal cost on the right hand side of the equation requires this interpretation.[8]

In order to estimate the system of marginal cost and input demand equations, Paris suggests the use of the generalized maximum entropy (GME) estimator. In case of positive degrees of freedom—as in the later application—other estimation criteria could have been employed as well. A motivation for applying GME nevertheless could be the possibility of including prior information by defining support points for both the parameters and the error terms. Since the GME approach implies that parameter estimates are (a) restricted to convex combinations of a priori specified finite supports and (b) drawn toward the simple average of the support as closely as the data constraints allow, the definition of support points should be motivated carefully.[9]

Paris motivates the support points for the parameters $\mathbf{Q}$, $\mathbf{H}$, and s by, in his view, plausible ranges and centers for the elements of $\mathbf{Q}$ (p. 1053). However, they are defined only *indirectly* in terms of supports for elements of a partitioned Cholesky factorization (p. 1052ff). Unfortunately, the specific and nonlinear structure of this decomposition implies support points for the elements of $\mathbf{Q}$, $\mathbf{H}$ and s that depend on the order of the crops. For illustration consider a simple 3×3 example for a part of Paris' Cholesky decomposition, $\mathbf{Q} = \mathbf{LDL}$, where we suppress the subscripts used in the text. Denoting the j, j'th element of $\mathbf{L}$ and $\mathbf{D}$ as $L_{jj'}$ and $D_{jj'}$, respectively, we have

$$(2) \qquad \mathbf{Q} = \begin{bmatrix} L_{11}^2 D_{11} & L_{11}D_{11}L_{21} & L_{11}D_{11}L_{31} \\ L_{11}D_{11}L_{21} & L_{21}^2 D_{11} + L_{22}^2 D_{22} & L_{21}D_{11}L_{31} + L_{22}D_{22}L_{32} \\ L_{11}D_{11}L_{31} & L_{21}D_{11}L_{31} + L_{22}D_{22}L_{32} & L_{31}^2 D_{11} + L_{32}^2 D_{22} + L_{33}^2 D_{33} \end{bmatrix}.$$

Suppose we had observations on three crops for a farm such that $(\lambda_{LPj} + c_j)/x_{Rj} = 4\,\forall j$. Based on the description on page 1053 we would then obtain the lower and upper support points for $\mathbf{L}$ and $\mathbf{D}$ as

$$(3) \qquad \mathbf{Z}_{\mathrm{L}} = \begin{bmatrix} 1 & 0 & 0 \\ (-4;4) & 1 & 0 \\ (-4;4) & (-4;4) & 1 \end{bmatrix} \quad \text{and}$$

$$\mathbf{Z}_{\mathrm{D}} = \begin{bmatrix} (0;8) & 0 & 0 \\ 0 & (0;8) & 0 \\ 0 & 0 & (0;8) \end{bmatrix}$$

which in turn imply the following lower and upper support points for the matrix $\mathbf{Q}$:[10]

(4)

$$\mathbf{Z}_{\mathrm{Q}} = \begin{bmatrix} (0;8) & (-32;32) & (-32;32) \\ (-32;32) & (0;136) & (-160;160) \\ (-32;32) & (-160;160) & (0;264) \end{bmatrix}.$$

The characteristics of the Cholesky factorization leads to different, order dependent, ranges for the elements of $\mathbf{Q}$, despite the obviously identical characteristics of the three crops in this case—a logical contradiction. Just changing the order of the crops would impact the value of the parameter estimates.[11] It can further be noticed that the units appropriate for $\mathbf{Q}$ and implied by the choice of support for

[8] Apart from these theoretical issues, there is also an empirical concern related to the cost function and how it is used by Paris. The parameter matrix $\mathbf{Q}$ implies "scale-variant" point elasticities of supply. As a consequence, for example, twenty identical farms would not have the same aggregate supply response as an aggregate model with the same parameters. Although many empirical models show this characteristic, the simultaneous use of farm models and aggregate model renders it a problematic choice. The error terms at farm level are biased by this specification. Some scaling of $\mathbf{Q}$ (see, e.g., Heckelei and Britz) or other functional forms would be appropriate in this case.

[9] This is especially true for positive degrees of freedom where supports on parameters would not even be required for consistent parameter estimation of the linear model (the proof in Mittelhammer and Cardell can be changed to accommodate infinite parameter supports, which was verified by personal communication with the authors).

[10] The support points for $\mathbf{Q}$ in (4) represent the minimum and maximum possible value for each element depending on the range of the elements in $\mathbf{L}$ and $\mathbf{D}$. The Cholesky factorization certainly puts restrictions on the admissible combination. For example, not all elements of $\mathbf{Q}$ could be at their lowest possible value at the same time.

[11] The comparatively small support ranges in the first row/column might explain the small absolute estimates for the first crop "sugar beet" in table 3.

elements of $\mathbf{L}$ and $\mathbf{D}$, [\$/weight^2], are not preserved in the transformation.

The support point problem of this approach is easily overcome: support points and associated probabilities can be defined for the elements of $\mathbf{Q}$ directly. The GME program would just require the classic Cholesky factorization of the form $\mathbf{Q} = \mathbf{LL}'$—or to return to the article's formulation—of the form $\left[\begin{smallmatrix}\mathbf{Q} & \mathbf{H}\\ \mathbf{H}' & s\end{smallmatrix}\right] = \mathbf{LL}'$ as a constraint to ensure proper curvature.[12]

Phase 3 and a Look across the Approach

The equilibrium problem (46)–(49) represents the final model to be used for simulation purposes. We agree with Paris insofar as we cannot find any underlying optimization model that would imply this system of equations for its optimal solutions. But how then is this model to be interpreted? Why should the conditions (47) and (48) hold in reality? Since $\boldsymbol{\beta}$ is required to be nonnegative, equation (47) ensures that the 'derived factor demand' (right hand side) is not smaller than the factor use $\mathbf{A}_n\mathbf{x}_n$ indicating that the average base year use per unit of output still applies. Equation (48) resembles a traditional marginal cost condition, but adds the cost of limiting resources under Leontief technology, $\mathbf{A}'_n\mathbf{y}_n$, to the derivative of the cost function. What rationale could possibly lead to such a combination of Leontief technology and a general cost function?

Apart from these additional questions related to the interpretation of the cost function, there is clearly an inconsistency between Phase 1 and Phases 2, 3. Whereas Phase 1 assumes profit maximization to estimate marginal cost and prices of limiting resources, Phases 2 and 3 employ some other (unknown) behavioral assumption.[13]

Application

Our final remarks concern the empirical application of SPEP to a cross-sectional sample of thirty-seven farms from the Italian Farm Accounting Data Network (p. 1055ff): (1) The sample includes just one year, 1995, so that the random components of observed prices and yields strongly influence the estimation results. (2) Prices for outputs differ considerably between the farms—the range for tomatoes is, for example, close to a factor of three. This casts additional doubt on the approach of using average sample values for missing data. (3) It is unclear if the sample was restricted a priori to farms producing the eight included field crops, or if information for all other production activities was simply eliminated. (4) The representation of the common agricultural policy (CAP) ignores important provisions: (*a*) The production of sugar beets is subject to a farm-specific sales quota, which is missing in the model; (*b*) whereas premiums for set-aside are introduced, the additional land requirement to fulfil set-aside obligation is not accounted for, which overestimates the profitability of cereals and oilseeds; (*c*) further complications arise from the fact that small producers (less then 80 tons of cereal equivalents produced) are exempt from obligatory set-aside, but receive lower premiums. Table 1 suggests that such farms are included in the data but only an (apparently) average set-aside rate of 5% is used. The inaccurate representation of the CAP alone casts doubt on the usefulness of the model for policy analysis—even setting aside the other considerable conceptual and methodological problems mentioned above.

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[12] For examples of this approach see Heckelei and Britz or Heckelei and Wolff.

[13] We cannot ourselves offer a satisfactory solution for the SPEP model, but Heckelei and Wolff demonstrate how explicit optimization models can be estimated without any employment of a Phase 1.